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EFFECT OF CIRCUMFERENTIAL NONUNIFORMITY OF FLOW DIRECTION ON RESONANCE VIBRATIONS OF COMPRESSOR ROTOR BLADES

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EFFECT OF CIRCUMFERENTIAL NONUNIFORMITY OF FLOW DIRECTION ON RESONANCE VIBRATIONS OF COMPRESSOR ROTOR BLADES

V. A. Kulagina

ABSTRACT: Estimation of the effect of a circumferential nonuniformity of the flow direction in front of a compressor wheel on the amplitude of resonance vibrations of the blades. The effect of the boundary conditions behind the wheel on the intensity of the flow spread in the inlet section of the compressor and on the blade vibrations is shown.

Until now the circumferential nonuniformity of flow direction in front of /96* a rotor operating in a flow with circumferentially nonuniform velocity distribution was not taken into account in determination of the amplitude of resonance vibrations of the axial compressor vanes. Meanwhile, theoretical works [4, 5] and experimental research [1] indicate that there is rapid spreading of the flow in front of the working compressor wheel.

An attempt is made in this work to consider and evaluate the effect of nonuniformity of the flow direction on the amplitude of resonance vibrations of the vanes. The problem is solved for low flow velocities in stationary, linear statement and is the logical continuation of previous works [2, 3].

Suppose we have a working axial compressor wheel as illustrated in Figure 1. Its gas dynamic characteristics are given by the dependences of the compression ratio $\pi_{\dot{c}} = p_2/p_1$ and theoretical head coefficient $\overline{H}_t$ on the air flow factor $\overline{c}_{1a}$ at calculated angle α_1.

In the compressor intake "at infinity" there is known circumferential nonuniformity of flow velocity; we represent flow nonuniformity in the intake into the working wheel as a sum of the nonuniformity of the velocity arriving from "infinity" and circumferential nonuniformity of static pressure, flow velocity and direction induced by the compressor impeller.

*Numbers in the margin indicate pagination in the foreign text.

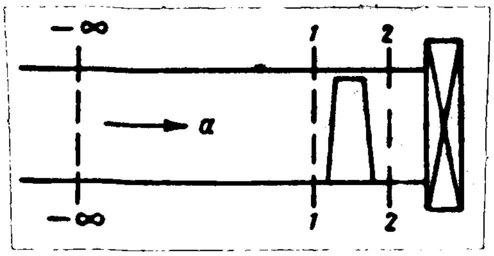

Figure 1. Flow Section of a Compressor.

We will assume that spreading of the flow occurs entirely within the intake part of the compressor, and that the flow does not spread in its vane section. It has been shown [4] that section assumption corresponds best of all to the physical pattern of the phenomenon. /97

We will assume that the angle of exit of the flow from the impeller in relative motion β_2 is constant with respect to circumference and that we know the slope of the gas-dynamic characteristic of the exhaust duct of the compressor, given in the form of the parameter

$$K = \frac{\delta p_2}{\delta c_{2a}} .$$

The compressor operates at the circumferential velocity at which resonance vibrations of the impeller vanes are stimulated according to the first flexure configuration, i.e., the frequency of natural vibrations ω_v of the vanes is a multiple of the revolution frequency of the compressor rotor $\omega_v = s\omega_c = s2\pi n_{sec}$. The vibrations of the vanes are harmonic:

$$x = x_0 \sin(\omega_v t \pm \beta_v);$$

Displacements of a vane in the presence of vibrations is illustrated in Figure 2. All vanes of the impeller have the same frequency of natural vibrations.

In the interest of simplifying further calculations we will introduce, instead of the gas-dynamic characteristic, the function $\varphi(\bar{c}_{1a}, \dot{\bar{x}}_a, \alpha_1)$, the rise of air pressure in an impeller with vibrating vanes:

$$(p_2 - p_1)/p_1 = (u + x_u)\varphi(\bar{c}_{1a}, \dot{\bar{x}}_a, \alpha_1),$$

and also the function $\psi(\bar{c}_{1a}, x_a, \alpha_1)$ — dimensionless power coefficient:

$$\psi(\bar{c}_{1a}, \dot{\bar{x}}_a, \alpha_1) = (\bar{c}_{1a} + \dot{\bar{x}}_a)\bar{H}_t(c_{1a}, \dot{\bar{x}}_a, \alpha_1).$$

Here $\dot{x}_a$, $\dot{x}_u$ are the components of the rate of vibrations of the vane with respect to the coordinate axes (see Figure 2): /98

$$\dot{x}_a = \dot{x}/u \cos\vartheta; \quad \dot{x}_u = -\frac{\dot{x}}{u}\sin\vartheta.$$

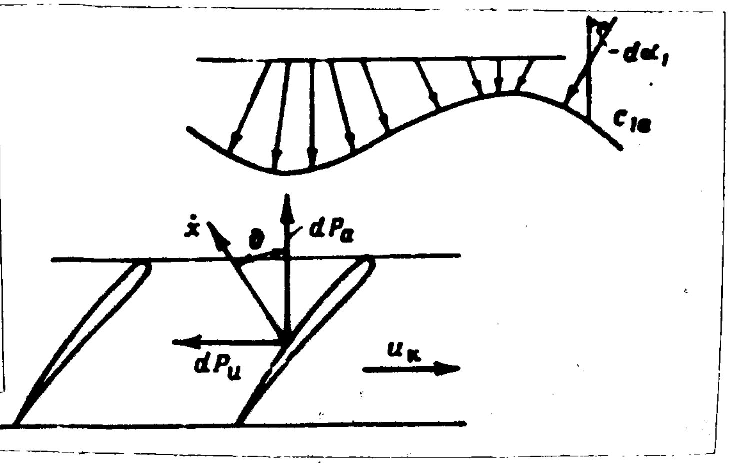

Figure 2. Displacements of Vane During Vibration.

We will first determine the parameters of circumferential nonuniformity of flow in front of and behind the compressor.

At infinity in front of the compressor the distribution of the flow velocity with respect to circumference is known:

$$c_{\infty a} = c_{\infty a\ av} + \sum_n \varepsilon_{nc} \cos n\theta + \varepsilon_{ns} \sin n\theta; \tag{1}$$

there is no nonuniformity of static pressure:

$$p_\infty = \text{const}; \tag{2}$$

flow direction is axial:

$$\alpha_\infty = \pi/2. \tag{3}$$

The relative magnitude of velocity nonuniformity in the compressor intake is determined by the nonuniformity of the total pressure loss coefficient:

$$\frac{c_\infty - c_{\infty\ av}}{c_{\infty\ av}} = \frac{\Delta c_\infty}{c_{\infty\ av}} = \frac{\Delta p^*_{in}}{M^2_{in}} = -\Delta\zeta_{in}$$

and at low flow velocities it does not depend on the number M_{in}.

In front of the impeller, in addition to velocity nonuniformity, is also compressor-induced circumferential flow nonuniformity. We will define it as follows:

$$c'_a(\theta,\ a);\ c_u(\theta,\ a);\ p'(\theta,\ a).$$

The flow parameters in the compressor intake will be:

$$c_a = c_{a\ av} + \sum_n \varepsilon_{nc} \cos n\theta + \varepsilon_{ns} \sin n\theta + c'_a; \tag{4}$$

$$p = p_{av} + p'; \tag{5}$$

$$c_u = c'_u. \tag{6}$$

We will denote

$$\frac{c_a = c_{a\ av}}{c_{a\ av}} = \delta c_a;\quad \frac{p - p_{av}}{p_{av}} = \delta p;\quad \frac{c'_u}{c_{a\ av}} = -d\alpha = -\frac{\pi}{2}\,\delta\alpha.$$

To find the flow in the compressor intake, i.e., to determine the functions /99
c'_a, p', c'_u, we will solve the equations of motion similarly as is done in [4]. We will assume that the flow moves along the cylindrical surfaces and that flow perturbations are small. In this case perturbed flow is potential. The equations of motion and continuity of the perturbed flow will have the following form:

$$r c_{a\ av} \frac{\partial c'_u}{\partial a} = -\frac{1}{\rho}\frac{\partial p'}{\partial \theta}; \tag{7}$$

$$c_{a\ av} \frac{\partial c'_a}{\partial a} = -\frac{1}{\rho}\frac{\partial p'}{\partial a}; \tag{8}$$

$$\frac{1}{r}\frac{\partial c'_u}{\partial \theta} + \frac{\partial c'_a}{\partial a} = 0. \tag{9}$$

The solution of the equations is found in the form of the series

$$c'_a = \sum_n z_{nc}(a)\cos n\theta + z_{ns}(a)\sin n\theta. \tag{10}$$

To determine the functions $z_n(a)$ we obtain the differential equation

$$\frac{r^2}{n^2}\frac{d^3 z_n}{d_a{}^3} - \frac{dz_n}{da} = 0 \tag{11}$$

with boundary conditions:
where $a = -\infty$ we have $c'_a = 0$; $z_{nc} = z_{ns} = 0$.
Solving equation (11), we obtain

$$z_{nc} = a_{nc}e^{na/r};\ z_{ns} = a_{ns}e^{na/r};\ a_{nc},\ a_{ns} — \text{constants.}$$

From equations (7)-(9), considering the above introduced definitions, we obtain the following expressions for circumferential nonuniformity of flow in section 1-1 directly in front of the impeller (flow a = 0);

$$\delta c_{1a} = \sum_n (\bar{\varepsilon}_{nc} + \bar{a}_{nc})\cos n\theta + (\bar{\varepsilon}_{ns} + \bar{a}_{ns})\sin n\theta; \tag{12}$$

$$\delta p_1 = -b\sum \bar{a}_{nc}\cos n\theta + \bar{a}_{ns}\sin n\theta, \tag{13}$$

where

$$b = \frac{\rho_1 c^2_{1a\ av}}{p1} \approx \kappa M_1^2;\ \bar{a}_{nc} = \frac{a_{nc}}{c_{a\ av}};\ \bar{a}_{ns} = \frac{a_{ns}}{c_{a\ av}};$$

$$\frac{c'_u}{c_{u\ av}} = -\frac{\pi}{2}\,\delta\alpha_1 = \sum -\bar{a}_{nc}\sin n\theta + \bar{a}_{ns}\cos n\theta. \tag{14}$$

The values $\bar{a}_{nc}$ and $\bar{a}_{ns}$ still remain unknown.

We will now examine flow in the vane section of the compressor. Since we assumed that the flow did not spread in the compressor, to analyze flow in /100 section 1-2 of the compressor (see Figure 1) we will use the equation of unidimensional fluid flow, similarly as done in [3]. For this section we may write three equations in finite differences.

From the equation of continuity we obtain equation

$$\delta p_1 + \delta c_{1a} = \delta p_2 + \delta c_{2a}. \tag{15}$$

For the dependence of π_c — compression ratio in an impeller with vibrating vanes — on $\bar{c}_{1a}$, $\dot{\bar{x}}_a$, α_1, u we obtain

$$\frac{\pi_c}{\pi_c - 1}(\delta p_2 - \delta p_1) = \delta u + \frac{\delta\varphi}{\delta\bar{c}_{1a}}(\delta c_{1a} - \delta u) + \frac{\delta\varphi}{\delta\dot{\bar{x}}_a}\dot{\bar{x}}_a + \frac{\delta\varphi}{\delta\alpha_1}\delta\alpha_1,$$

or

$$\frac{\pi_c}{\pi_c - 1}(\delta p_2 - \delta p_1) = \frac{\delta\varphi}{\delta c_{1a}}\delta c_{1a} + \frac{\bar{c}_{1a}}{\bar{H}_t}\frac{\alpha_1}{\sin^2\alpha_1}\delta\alpha_1 + \dot{\bar{x}}\left(\xi\varphi\frac{\delta\varphi}{\delta\bar{c}_{1a}} - 2\sin\vartheta\right), \tag{16}$$

where

$$\xi_\varphi = \sin\vartheta + \frac{\cos\vartheta(\delta\varphi/\dot{\bar{x}}_a)}{\bar{c}_{1a}(\delta\varphi/\delta\bar{c}_{1a})} = \sin\vartheta + \frac{\cos\vartheta}{\bar{c}_{1a}}\left[1 + \frac{\bar{c}_{1a}\cot\alpha_1}{\bar{H}_t(\delta\varphi/\delta\bar{c}_{1a})}\right].$$

The derivative $\delta\varphi/\delta\bar{c}_{1a} = d\pi_c/(d\bar{c}_{1a})\ (\bar{c}_{1a})/(\pi_c - 1)$ can be found graphically according to the given characteristic of the impeller; in determining the derivatives $\delta\varphi/\delta\dot{\bar{x}}_a$ and $\delta\varphi/\delta\alpha_1$ it was assumed that they are proportional to the values $\delta\bar{H}_t/\delta\dot{\bar{x}}_a$ and $\delta\bar{H}_t/\delta\alpha_1$, respectively.

The approximate equation

$$\bar{H}_t = 1 - (\bar{c}_{2a} + \dot{\bar{x}}_a)\cot\beta_2 - \bar{c}_{1a}\cot\alpha_1. \tag{17}$$

was used for the theoretical head coefficient $\bar{H}_t$. The boundary condition in which is employed the slope of the compressor exhaust duct characteristic gives us the third equation:

$$\delta p_2 = K\delta c_{2a}. \tag{18}$$

From expressions (15) and (18) we obtain

$$\delta p_2 = K/(K+1)(\delta p_1 + \delta c_{1a}); \tag{19}$$

$$\delta c_{2a} = 1/(K+1)(\delta p_1 + \delta c_{1a}). \tag{20}$$

Equation (16) now acquires the following form: /101

$$-\frac{\pi_c}{\pi_c - 1}\frac{1}{K+1}\delta p_1 + \left(\frac{\pi_c}{\pi_c - 1}\frac{K}{K+1} - \frac{\delta\varphi}{\delta\bar{c}_{1a}}\right)\delta\bar{c}_{1a} - \frac{\bar{c}_{1a}}{\bar{H}_t}\frac{\alpha_1}{\sin^2\alpha}\delta\alpha_1 - \dot{\bar{x}}\left(\xi\varphi\frac{\delta\varphi}{\delta\bar{c}_{1a}} - 2\sin\vartheta\right) = 0 \tag{21}$$

and can be used for joining the solutions found for compressor segments ∞ - 1 and 1-2 (see Figure 1) and for determining the constants $\bar{a}_{nc}$ and $\bar{a}_{ns}$.

We will denote

$$-\frac{\pi_c}{\pi_c - 1}\frac{1}{K+1} = A; \quad \frac{\pi_c}{\pi_c - 1}\frac{K}{K+1} - \frac{\delta\varphi}{\delta\bar{c}_{1a}} = B;$$

$$\frac{\bar{c}_{1a}}{\bar{H}_t}\frac{1}{\sin^2\alpha} = C; \quad \xi\varphi\frac{\delta\varphi}{\delta\bar{c}_{1a}} - 2\sin\vartheta = D.$$

Equation (21) acquires the following form:

$$A\delta p_1 + B\delta c_{1a} - C\alpha_1\delta\alpha_1 - L\dot{\bar{x}} = 0. \tag{22}$$

We will seek the constants a_{sc}, a_{ss}, corresponding to a harmonic of the order s, exciting resonance vibrations of the vanes (n = s). The flowing coordinate in the circumferential direction $\theta = 2\pi n_{sec} t = \omega_c t$ can be expressed through the frequency of vane vibrations:

$$n\theta = s\theta = s\omega_c t = \omega_v t.$$

The displacements of the vane in the presence of vibrations are

$$x = x_0 \sin(\omega_v t + \beta_v),$$

and the rate of vibrations is

$$\dot{\bar{x}} = \frac{x_0 \omega_v}{u} \cos(\omega_v t = \beta_v); \quad \frac{x_0 \omega_v}{u} = \dot{\bar{x}}_0.$$

The phase of vane vibrations β_v is still unknown.

By substituting the expressions for δp_1, δc_{1a}, $\delta \alpha_1$ into equation (22) we find the following equation for the s-th harmonic causing the vibrations:

$$\begin{gathered}\cos\omega_v t[(-Ab + B)\bar{a}_{sc} + B\bar{\varepsilon}_{sc} + C\bar{a}_{ss} - D\dot{\bar{x}}_0 \cos\beta_v] + \\ + \sin\omega_v t[-C\bar{a}_{sc} - (Ab + B)\bar{a}_{ss} + B\bar{\varepsilon}_{ss} + D\dot{\bar{x}}_0 \sin\beta_v] = 0.\end{gathered} \tag{23}$$

Equating to zero the coefficients for $\cos\omega_v t$ and $\sin\omega_v t$, we obtain two equations for determining unknown $\bar{a}_{sc}$ and $\bar{a}_{ss}$.

Solving these equations we obtain /102

$$\bar{a}_{sc} = \frac{-\bar{\varepsilon}_{sc} B(-Ab + B) + \bar{\varepsilon}_{ss} BC \frac{\alpha_1}{(\pi/2)} - \dot{\bar{x}}_0 \cos\beta_v D(-Ab + B) + \dot{\bar{x}}_0 \sin\beta_v DC \frac{\alpha_1}{(\pi/2)}}{(-Ab + B)^2 + C^2 \left(\frac{\alpha_1}{(\pi/2)}\right)^2}; \tag{24}$$

$$\bar{a}_{ss} = \frac{-\bar{\varepsilon}_{sc} BC \dfrac{\alpha_1}{(\pi/2)} - \bar{\varepsilon}_{ss} B\ (-\ Ab + B) + \dot{\bar{x}}_0 \cos\beta_v DC \dfrac{\alpha_1}{(\pi/2)} - -\dot{\bar{x}}_0 \sin\beta_1 D(\ -Ab + B)}{(-Ab + B)^2 + C^2 \left[\dfrac{\alpha_1}{(\pi/2)}\right]^2} . \tag{25}$$

For all nonresonating harmonics $n \neq s$, the vibrations of the vanes $\dot{x} = 0$.

In the case when the initial velocity nonuniformity (at infinity) has axial symmetry velocity distribution with respect to circumference is described by a series in cosigns and all coefficients $\varepsilon_{ns} = 0$. In the remainder of the discussion, first simplicity, we will examine such a case.

We will replace A, B, C, D by their values.

Considering that the value $(\pi_c - 1)$ is small at low flow velocities and its squares can be ignored, we obtain

$$\bar{a}_{sc} = \frac{-\bar{\varepsilon}_{sc}\left(\dfrac{K}{K + 1} - \dfrac{\pi_c - 1}{\pi_c} \dfrac{\delta\varphi}{\delta\bar{c}_{1a}}\right) + \dot{\bar{x}}_0 \cos\beta_v \dfrac{\pi_c - 1}{\pi_c}\left(\xi_\varphi \dfrac{\delta\varphi}{\delta\bar{c}_{1a}} - 2\sin\vartheta\right)}{\dfrac{K + b}{K + 1} - \dfrac{\pi_c - 1}{\pi_c} \dfrac{\delta\varphi}{\delta\bar{c}_{1a}}}; \tag{26}$$

$$a_{ss} = -\frac{\bar{\varepsilon}_{sc} \dfrac{\pi_c - 1}{\pi_c} \dfrac{\bar{c}_{1a}}{\bar{H}_t} \dfrac{1}{\sin^2\alpha_1} \dfrac{\alpha_1}{(\pi/2)} \dfrac{K}{K + 1}}{\left(\dfrac{K + b}{K + 1} - \dfrac{\pi_c - 1}{\pi_c} \dfrac{\delta\varphi}{\delta\bar{c}_{1a}}\right)^2}$$

$$- \frac{\dot{\bar{x}}_0 \sin\beta_v \dfrac{\pi_c - 1}{\pi_c}\left(\xi_\varphi \dfrac{\delta\varphi}{\delta\bar{c}_{1a}} - 2\sin\vartheta\right)}{\dfrac{K + b}{K + 1} - \dfrac{\pi_c - 1}{\pi_c} \dfrac{\delta\varphi}{\delta\bar{c}_{1a}}} . \tag{27}$$

We may now write the expression for flow nonuniformity in front of the impeller, using equations (12), (13) and (14).

For the case when $K \geqslant 0$, we obtain

$$\delta p_1 = \frac{Kb}{K+b} \sum_n \bar{\varepsilon}_{nc} \cos n\theta; \tag{28}$$

$$\delta c_{1a} = \frac{b}{K+b} \sum_n \bar{\varepsilon}_{nc} \cos n\theta; \tag{29}$$ /103

$$-\delta\alpha_1 \frac{\pi}{2} = \frac{K}{K+b} \sum_n \bar{\varepsilon}_{nc} \sin n\theta, \tag{30}$$

and for the case $K = 0$

$$\delta p_1 = \frac{-b\dfrac{\pi_c - 1}{\pi_c}\dfrac{\delta\varphi}{\delta\bar{c}_{1a}}}{b - \dfrac{\pi_c - 1}{\pi_c}\dfrac{\delta\varphi}{\delta\bar{c}_{1a}}} \sum \bar{\varepsilon}_{nc} \cos n\theta; \tag{31}$$

$$\delta c_{1a} = \frac{b}{b - \dfrac{\pi_c - 1}{\pi_c}\dfrac{\delta\varphi}{\delta\bar{c}_{1a}}} \sum \bar{\varepsilon}_{nc} \cos n\theta; \tag{32}$$

$$-\delta\alpha_1 \frac{\pi}{2} = \frac{-\dfrac{\pi_c - 1}{\pi_c}\dfrac{\delta\varphi}{\delta\bar{c}_{1a}}}{b - \dfrac{\pi_c - 1}{\pi_c}\dfrac{\delta\varphi}{\delta\bar{c}_{1a}}} \sum \bar{\varepsilon}_{nc} \sin n\theta. \tag{33}$$

We will now examine the aerodynamic forces acting on a vane vibrating in a circumferentially nonuniform flow. For small flow velocities the component of aerodynamic force in the direction of the compressor axis can be expressed as follows:

$$P_a = F_v p_1(\pi_c - 1); \tag{34}$$

hence

$$\frac{dP_a}{F_v p_1} = \pi_c \delta p_2 - \delta p_1 + (\pi_c - 1)\delta F_v.$$

Considering the change of area δF_v, as a result of the out-of-phase vibrations of adjacent vanes, equates to $\delta F_v = -\bar{x} \sin \vartheta$ for small-order harmonics (see [3]), we obtain

$$\frac{dP_a}{F_v p_1} = (\pi_c - 1)\left[\frac{\pi_c}{\pi_c - 1}(\delta p_2 - \delta p_1) + \delta p_1 - \dot{\bar{x}} \sin \vartheta\right].$$

Using condition (16), we may write, finally:

$$\frac{dP_a}{F_v p_1} = (\pi_c - 1)\left[\delta p_1 + \frac{\delta\varphi}{\delta \bar{c}_{1a}}\, \delta c_{1a} = \frac{\bar{c}_{1a}}{\bar{H}_t}\, \frac{\alpha_1}{\sin^2\alpha}\, \delta\alpha_1 + \right.$$

$$\left. + \dot{\bar{x}}\left(\xi_\varphi \frac{\delta\varphi}{\delta \bar{c}_{1a}} - 3 \sin \vartheta\right)\right]. \tag{35}$$

The component of aerodynamic force in the circumferential direction is /104

$$P_u = F_v \rho_1 (u + \dot{\bar{x}}_u)^2 \psi(\bar{c}_{1a}, \dot{\bar{x}}_a, \alpha_1),$$

where

$$\psi(\bar{c}_{1a}, \dot{\bar{x}}_a, \alpha_1) = (\bar{c}_{1a} + \dot{\bar{x}}_a)\bar{H}_t(\bar{c}_{1a}, \dot{\bar{x}}_a, \alpha_1). \tag{36}$$

Using expression (17) we obtain

$$\frac{dP_u}{F_v pt} = (\pi_c - 1)\frac{\bar{c}_{1a}}{\eta\tau}\left[\delta p_1 + \frac{\delta\psi}{\delta\bar{c}_{1a}}\delta c_{1a} + \frac{\bar{c}_{1a}}{\bar{H}_\tau}\frac{\alpha_1}{\sin^2\alpha_1}\delta\alpha_1 + \right.$$

$$\left. + \dot{x}\left(\xi_\psi \frac{\delta\psi}{\delta\bar{c}_{1a}} - 3\sin\vartheta\right)\right]; \qquad (37)$$

here

$$\xi_\psi = \sin\vartheta + \frac{\cos\vartheta}{\bar{c}_{1a}}\frac{(\delta\psi/\delta\dot{x}_a)}{(\delta\psi/\delta\bar{c}_{1a})} = \sin\vartheta + \frac{\cos\vartheta}{\bar{c}_{1a}}\left[1 + \frac{\bar{c}_{1a}\cot\alpha_1}{\bar{H}_\tau(\delta\psi/\delta\bar{c}_{1a})}\right].$$

The variable aerodynamic force acting in the direction of vane vibrations, can be expressed as follows (Figure 2):

$$dP_\vartheta = dP_a\cos\vartheta + dP_u\sin\vartheta;$$

$$\frac{dP_\vartheta}{F_v p_1} = (\pi_c - 1)\left\{\delta p_1\left(\cos\vartheta + \frac{\bar{c}_{1a}}{\eta\tau}\sin\vartheta\right) + \delta\alpha_1\frac{\bar{c}_{1a}\alpha_1}{\bar{H}_\tau\sin^2\alpha}\left(\cos\vartheta + \right.\right.$$

$$\left. + \frac{\bar{c}_{1a}}{\eta\tau}\sin\vartheta\right) + \delta c_{1a}\left(\frac{\delta\psi}{\delta\bar{c}_{1a}}\cos\vartheta + \frac{\bar{c}_{1a}}{\eta\tau}\frac{\delta\psi}{\delta\bar{c}_{1a}}\sin\vartheta\right) + \dot{x}\left[\left(\xi_\varphi\frac{\delta\varphi}{\delta\bar{c}_{1a}} - \right.\right.$$

$$\left.\left.\left. -3\sin\vartheta\right)\cos\vartheta + \frac{\bar{c}_{1a}}{\eta\tau}\left(\xi_\psi\frac{\delta\psi}{\delta\bar{c}_{1a}} - 3\sin\vartheta\right)\sin\vartheta\right]\right\}. \qquad (38)$$

We will substitute expressions (28)-(30) and (31)-(33) into the equation for the force dP_ϑ and extract only the resonating harmonic of the force ($n = s$, $n\theta = \omega_v z$).

Then for $K \gg 0$ we obtain

$$\frac{dP_\vartheta}{F_v p_1} = (\pi_c - 1)\Big\{\bar{\varepsilon}_{nc}\cos\omega_v t\Big[\frac{Kb}{K+b}\left(\cos\vartheta + \frac{\bar{c}_{1a}}{\eta\tau}\sin\vartheta\right) +$$

$$+\frac{b}{K+b}\left(\frac{\delta\varphi}{\delta\bar{c}_{1a}}\cos\vartheta + \frac{\bar{c}_{1a}}{\eta\tau}\ \frac{\delta\psi}{\delta\bar{c}_{1a}}\ \sin\vartheta\right)\Big] - \bar{\varepsilon}_{nc}\sin\omega_v t\ \frac{K}{K+b}\ \frac{\bar{c}_{1a}}{\bar{H}_\tau}\times$$

$$\times\frac{1}{\sin^2\alpha_1}\ \frac{\alpha_1}{(\pi/2)}\left(\cos\vartheta + \frac{\bar{c}_{1a}}{\eta\tau}\sin\vartheta\right) + \dot{x}\Big[\left(\xi_\varphi\ \frac{\delta\varphi}{\delta\bar{c}_{1a}} - 3\sin\vartheta\right)\cos\vartheta +$$

$$+\frac{\bar{c}_{1a}}{\eta\tau}\left(\xi_\psi\ \frac{\delta\psi}{\delta\bar{c}_{1a}} - 3\sin\vartheta\right)\sin\Big]\Big\}; \qquad (39)$$

and for $K = 0$ /105

$$\frac{dP_\vartheta}{F_v p_1} = (\pi_c - 1)\Bigg\{\bar{\varepsilon}_{nc}\cos\omega_v t\left[\frac{-b\ \dfrac{\pi_c - 1}{\pi_c}\ \dfrac{\delta\psi}{\delta\bar{c}_{1a}}}{b - \dfrac{\pi_c - 1}{\pi_c}\ \dfrac{\delta\varphi}{\delta\bar{c}_{1a}}}\left(\cos\vartheta + \frac{\bar{c}_{1a}}{\eta\tau}\sin\vartheta\right) +\right.$$

$$\left.+\frac{b}{b - \dfrac{\pi_c - 1}{\pi_c}\ \dfrac{\delta\varphi}{\delta\bar{c}_{1a}}}\left(\frac{\delta\varphi}{\delta\bar{c}_{1a}}\cos\vartheta + \frac{\bar{c}_{1a}}{\eta\tau}\ \frac{\delta\psi}{\delta\bar{c}_{1a}}\sin\vartheta\right)\right] -$$

$$-\bar{\varepsilon}_{nc}\sin\omega_v t\ \frac{\bar{c}_{1a}}{\bar{H}_\tau}\ \frac{1}{\sin\alpha_1}\ \frac{\alpha_1}{(\pi/2)}\ \frac{\dfrac{\pi_c - 1}{\pi_c}\ \dfrac{\delta\varphi}{\delta\bar{c}_{1a}}}{b - \dfrac{\pi_c - 1}{\pi_c}\ \dfrac{\delta\varphi}{\delta\bar{c}_{1a}}}\Big(\cos\vartheta +$$

$$+\frac{\bar{c}_{1a}}{\eta\tau}\sin\vartheta\Big) + \dot{x}\Big[\left(\xi_\varphi\ \frac{\delta\varphi}{\delta\bar{c}_{1a}} - 3\sin\vartheta\right)\cos\vartheta + \qquad (40)$$

$$+\frac{\bar{c}_{1a}}{\eta\tau}\left(\xi_\psi \frac{\delta\psi}{\delta\bar{c}_{1a}} - 3\sin\vartheta\right)\sin\vartheta\Bigg]\Bigg\} . \qquad (40)$$

For resonance vibrations of vanes, ignoring the forces of mechanical damping, we may write

$$dP_\vartheta = 0.$$

Consequently,

$$\bar{\varepsilon}_{nc}\cos\omega_v t C_{B_1} + \bar{\varepsilon}_{nc}\sin\omega_v t C_{B_2} + \frac{x_0\omega_v}{u}\cos(\omega_v t + \beta_v)C_D = 0. \qquad (41)$$

We rewrite equation (41) in the form

$$\bar{\varepsilon}_{nc}\sqrt{C_{B_1}^2 + C_{B_2}^2}\cos(\omega_v t + \beta_B) = -C_D\frac{x_0\omega_v}{u}\cos(\omega_v t + \beta_v); \qquad (42)$$

hence

$$x_0 = \frac{\left|\bar{\varepsilon}_{nc}\sqrt{C_{B_1}^2 + C_{B_2}^2}\right|}{-C_D}\,\frac{u}{\omega_v}; \qquad (43)$$

$$\beta_v = \beta_B = \text{arc tan} - \frac{C_{B_2}}{C_{B_1}} . \qquad (44)$$

In the absence of flow spread in front of the impeller

$$x_0 = \frac{\left|\bar{\varepsilon}_{nc}C_{B_1}\right|}{-C_D}\,\frac{u}{\omega_v} . \qquad (45)$$

Comparison of expressions (43) and (45) shows that the effect of circum- /106
ferential nonuniformity of flow direction in front of the impeller on vane resonant vibrations is manifested in a change of the coefficient of aerodynamic excitation. There is an additional term C_{B_2} which has considerable value. The so-called point of zero excitation, previously the object of attention [2]

and [3], where only the nonuniformity of axial velocity and static pressure was examined, vanishes. We will recall that the presence of the point of zero excitation is not supported by experimental data.

We will examine the limiting cases of the boundary conditions:

1) K = 0 Complete Equalization of Static Pressure Behind Impeller

$$C_{B_1} = \frac{-1.4M_1^2 \dfrac{\pi_c - 1}{\pi_c} \dfrac{\delta\varphi}{\delta\bar{c}_{1a}}}{1.4M_1^2 - \dfrac{\pi_c - 1}{\pi_c} \dfrac{\delta\varphi}{\delta\bar{c}_{1a}}} \left(\cos\vartheta + \frac{\bar{c}_{1a}}{\eta\tau}\sin\vartheta\right) +$$

$$+ \frac{1.4M_1^2}{1.4M_1^2 - \dfrac{\pi_c - 1}{\pi_c} \dfrac{\delta\varphi}{\delta\bar{c}_{1a}}} \left(\frac{\delta\varphi}{\delta\bar{c}_{1a}}\cos\vartheta + \frac{\bar{c}_{1a}}{\eta\tau}\frac{\delta\psi}{\delta\bar{c}_{1a}}\sin\vartheta\right);$$

$$C_{B_2} = \frac{\bar{c}_{1a}}{\bar{H}_\tau} \frac{1}{\sin^2\alpha_1} \frac{\alpha_1}{(\pi/2)} \frac{\dfrac{\pi_c - 1}{\pi_c} \dfrac{\delta\varphi}{\delta\bar{c}_{1a}}}{1.4M_1^2 - \dfrac{\pi_c - 1}{\pi_c} \dfrac{\delta\varphi}{\delta\bar{c}_{1a}}} \left(\cos\vartheta + \frac{\bar{c}_{1a}}{\eta\tau}\sin\vartheta\right);$$

$$C_D = \left(\xi_\varphi \frac{\delta\varphi}{\delta\bar{c}_{1a}} - 3\sin\vartheta\right)\cos\vartheta + \frac{\bar{c}_{1a}}{\eta\tau}\left(\xi_\psi \frac{\delta\psi}{\delta\bar{c}_{1a}} - 3\sin\vartheta\right)\sin\vartheta.$$

At the point where $C_{B_1} = 0$, the coefficient is $C_{B_1} \neq 0$ (if $\dfrac{\pi_c - 1}{\pi_c} \dfrac{\delta\varphi}{\delta\bar{c}_{1a}} = \dfrac{\bar{c}_{1a}}{\pi_c} \dfrac{d\pi_c}{d\bar{c}_{1a}} \neq 0$) and the amplitude of vane vibrations $x_0 \neq 0$.

2). $K = \infty$ No Nonuniformity of Velocity

$$C_{B_1} = 1.4M_1^2 \left(\cos\vartheta + \frac{\bar{c}_{1a}}{\eta\tau}\sin\vartheta\right);$$

$$C_{B_2} = \frac{\bar{c}_{1a}}{\bar{H}_\tau \sin^2\alpha_1}\,\frac{\alpha_1}{(\pi/2)}\left(\cos\vartheta + \frac{\bar{c}_{1a}}{\eta\tau}\sin\vartheta\right);$$

$$C_D = \left(\xi_\varphi \frac{\delta\varphi}{\delta\bar{c}_{1a}} - 3\sin\vartheta\right)\cos\vartheta + \frac{\bar{c}_{1a}}{\eta\tau}\left(\xi_\psi \frac{\delta\psi}{\delta\bar{c}_{1a}} - 3\sin\vartheta\right)\sin\vartheta.$$

The amplitude of vane vibrations is determined completely for flow spread /107
at the compressor inlet (coefficient C_{B_1}).

The results of the calculation are presented in Figure 3. The dependence of the relative (dimensionless) amplitude of resonance vibrations of the vanes on the air flow coefficient $\bar{c}_{1a}$ — is illustrated with and without consideration of the spreading of the flow in the compressor inlet or various parameters K. We see that allowance for the effect of nonuniformity of flow direction in front of the impeller substantially altered the path of the amplitude curves. The point of zero amplitude vanished and the results of the calculation thereupon approximated the experimental results.

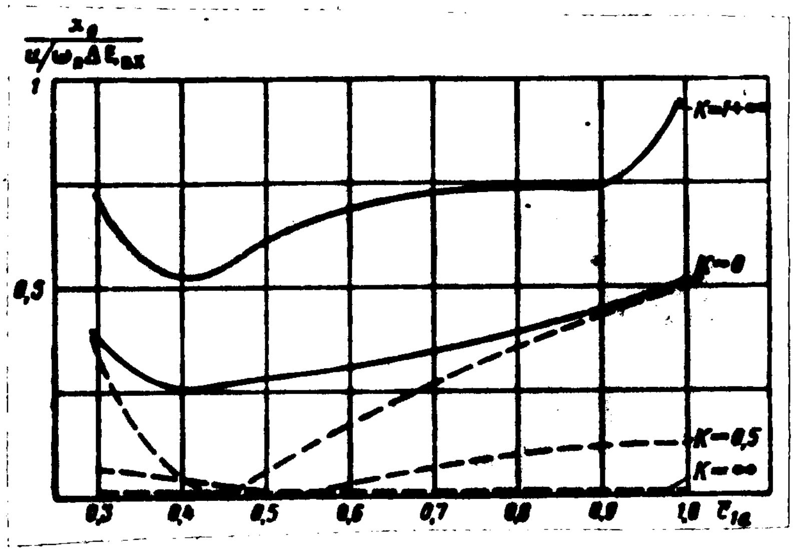

Figure 3. Dependence of Relative Amplitude of Resonance Vibrations of the Vanes on Their Flow Coefficient: ————, Calculation with allowance for flow spreading in inlet; ------, Calculation without allowance for spreading.

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